Multiobjective $H_2/H_\infty$ control design with regional pole constraints for damping power system oscillations

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This paper presents multiobjective $H_2/H_\infty$ control design with regional pole constraints for damping power system oscillations. The state feedback gain can be obtained by solving a linear matrix inequality (LMI) feasibility problem that robustly assigns the closed-loop poles in a prescribed LMI region. The proposed technique is illustrated with applications to the design of stabilizer for a typical single-machine infinite-bus (SMIB) power system. The LMI-based control ensures adequate damping for widely varying system operating conditions. The simulation results illustrate the effectiveness and robustness of the proposed stabilizer.

Keywords: multiobjective $H_2/H_\infty$ control, regional pole constraints, linear matrix inequality, power system stabilizer, robust control

1. Introduction

Power systems are usually large nonlinear systems, which are often subject to low frequency oscillations when working under some adverse loading conditions. The oscillation may sustain and grow to cause system separation if no adequate damping is available. To enhance system damping, the generators are equipped with power system stabilizers (PSSs) that provide supplementary feedback stabilizing signals in the excitation systems. PSSs extend the power system stability limit by enhancing the system damping of low frequency oscillations associated with the electromechanical modes [1]. Many approaches are available for PSS design, most of which are based either on classical control methods [1-3] or on intelligent control strategies [4-6].

However, as power systems are large nonlinear systems, it is impossible for the system to always run at the preselected operating conditions. When the system is
away from the specified operating point, the performance of the PSS will degenerate. Power systems continually undergo changes in the operating condition due to changes in the loads, generation and the transmission network resulting in accompanying changes in the system dynamics. In other words, the stabilizer should be robust to changes in the power system over its entire operating range.

In the last few years, robust control technique has been applied to power system controller design to guarantee robust performance and robust stability, due to uncertainty in plant parameter variations. Some of those efforts have been contributed to design robust controllers for PSS and/or FACTS devices using $H_\infty$ concept such as mixed-sensitivity [7-10]; $\mu$ -synthesis or structured singular value (SSV) [11] and $H_2$ norm concept such as LQG [12]. Normally, the problem is formulated as a weighted mixed-sensitivity design and solved by a Riccati approach. In addition, robust $H_\infty$ design being essentially a frequency domain approach does not provide much control over the transient behavior and closed-loop pole location. Robust pole placement stabilizer design using linear matrix inequalities (LMIs) has been presented in Ref. [13], where the feedback gain matrix is obtained as the solution of a linear matrix inequality expressing the pole region constraints for polytopic plants.

Design methods based on the $H_\infty$ norm of the closed-loop transfer function have gained popularity, because unlike $H_2$ methods (best known as LQG), they offer a single framework in which to deal both with performance and robustness. On the other hand, since an $H_2$ cost function offers a more natural way of representing certain aspects of the system performance, improving the robustness of $H_2$ based design methods against perturbations of the nominal plant is a problem of considerable importance for practical applications [14]. In the robust $H_2$ approach, the controller is designed to minimize an upper bound on the worst- case $H_2$ norm for a range of admissible plant perturbations. Thus, a combination of $H_2$ control and $H_\infty$ control, called multiobjective or mixed $H_2/H_\infty$ control that minimized the $H_2$ norm of some closed-loop function subject to the $H_\infty$ norm constraint of another closed-loop function. Khargonekar et al. [18] considered state- and output-feedback problems of multiobjective $H_2/H_\infty$ control and gave efficient convex optimization
approach to solve the coupled nonlinear matrix Riccati equations.

With the development of numerical algorithms for solving linear matrix inequality (LMI) problems in the last 8 years, the LMI approach have emerged as a useful tool for solving a wide variety of control problems [16]. One of the advantages of linear matrix inequality (LMI) is mixing the time and frequency domain objectives. This paper proposes a multiobjective $H_2/H_\infty$ control design with regional pole constraints for damping power system oscillations based on linear matrix inequality. The efficiency of an LMI-based design approach as a practical design tool is illustrated with case study, including a typical single-machine infinite-bus (SMIB) power system.

The paper is organized as follows. A detailed description of the proposed design procedure is given in Section 2. Dynamic model of the power system is given in Section 3. In Section 4, simulation results are given for a typical single-machine infinite-bus (SMIB) power system to demonstrate the effectiveness of the proposed method. Conclusions are drawn in Section 5.

2. LMI-based multiobjective $H_2/H_\infty$ controller design

Stability is a minimum requirement for control system. However, in most practical situations, a good controller should also deliver sufficiently fast and well-damped time responses. A customary way to guarantee satisfactory transients is to place the closed-loop poles in a suitable region of the complex s-plane.

This section discusses state feedback synthesis with a combination of multiobjective $H_2/H_\infty$ performance and pole assignment specifications. Here, the closed-loop poles are required to lie in some LMI region $D$ contained in the left-half plane. Unconstrained multiobjective $H_2/H_\infty$ synthesis is considered in [18], where an LMI-based synthesis procedure is proposed. Excellent background material on LMI may be found in [15].

2.1. Introduction of linear matrix inequality

A wide variety of problems in control theory and system can be reduced to a handful of standard convex and quasi-convex optimization problems that involve
linear matrix inequalities (LMIs), that is constraints of the form [15]:

\[ F(x) = F_0 + \sum_{i=1}^{m} x_i F_i \succ 0 \tag{1} \]

where \( x \in \mathbb{R}^m \) is the variable, and \( F_i = F_i^T \in \mathbb{R}^{m \times m} \) are given. The set \( \{ x \mid F(x) \succ 0 \} \) is convex, and need not have smooth boundary.

When the matrices \( F_i \) are diagonal, the LMI \( F(x) \succ 0 \) is just a set of linear inequalities. Nonlinear (convex) inequalities are converted to LMI form using Schur complements. The basic idea is as follows:

\[
\begin{bmatrix}
Q(x) & S(x) \\
S(x)^T & R(x)
\end{bmatrix} > 0
\]

where \( Q(x) = Q(x)^T, R(x) = R(x)^T \), and \( S(x) \) depend affinely on \( x \), is equivalent to

\[
R(x) > 0, \quad Q(x) - S(x)R(x)^{-1}S(x)^T > 0 \tag{3}
\]

In other words, the set of nonlinear inequalities Eq. (3) can be represented as the LMI Eq. (2).

Two standard LMI optimization problems are of interest:

1. LMI feasibility problem. Given an LMI \( F(x) \succ 0 \), the corresponding LMI feasibility problem is to find \( x^{feas} \) such that \( F(x^{feas}) \succ 0 \) or determine that the LMI is infeasible.

2. Semi-definite Programming problem (SDP). An SDP requires the minimization of a linear objective subject to LMI constraints:

\[
\text{Minimize} \quad c^T x \\
\text{Subject to} \quad F(x) \succ 0
\]

where \( c \) is a real vector, and \( F \) is a symmetric matrix that depends affinely on the optimization variable \( x \). This is a convex optimization problem.

Both these problems can be numerically solved vary efficiently, using currently available software [16,17].

2.2. LMI formulation for multiobjective \( H_2/H_\infty \) performance

Consider the linear plant \( P \) with input \( u \), disturbance \( w \), performance output \( z_\infty \) and \( z_2 \), the measurement signal \( x \). The input is generated by state feedback, using the controller \( K \). The signal \( z_\infty \) is the performance associated with the \( H_\infty \)
constraint, the signal $z_2$ is the performance associated with the $H_2$ criterion. The state space representation of the controlled system can be written as follows:

$$
\dot{x} = Ax + B_1u + B_2u \\
z_\infty = C_1x + D_{12}u \\
z_2 = C_2x + D_{22}u
$$

where all the matrices are constant real matrices of appropriate dimension. The illustration of the controlled system is shown in Fig.1.

![Fig. 1. Generalized plant](image)

After substitution of the state feedback controller $u = Kx$ into Eq. (5), the closed-loop system becomes

$$
\dot{x} = (A + B_2 K)x + B_1 w \\
z_\infty = (C_1 + D_{12} K)x \\
z_2 = (C_2 + D_{22} K)x
$$

Let $T_{z,w}$ and $T_{z_2,w}$ be the closed-loop transfer matrices from the generalized disturbance $w$ to the performance output $z_\infty$ and $z_2$, respectively:

$$
T_{z,w}(s) = \begin{bmatrix} A + B_2 K & B_1 \\ C_1 + D_{12} K & 0 \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl1} & 0 \end{bmatrix}
$$

$$
T_{z_2,w}(s) = \begin{bmatrix} A + B_2 K & B_1 \\ C_2 + D_{22} K & 0 \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl2} & 0 \end{bmatrix}
$$

The goal of multiobjective $H_2/H_\infty$ control is to find an internally stabilizing controller $K$ that minimizes the $H_2$ performance, $\|T_{z_2,w}\|_2$, subject to the
$H_\infty$ performance, $\left| T_{z,w} \right|_\infty < \gamma$ and places the closed-loop poles in some LMI stability region $D$ that will be explained in the next subsection. In this subsection, pure $H_2$ and $H_\infty$ synthesis are not given. For proofs and more details, see [19,20].

We are now ready to give tractable necessary and sufficient conditions for solving the following multiobjective $H_2/H_\infty$ problem:

$$\min \ trace(C_{cl2} P C_{cl2}^T)$$

$$\text{s.t. } \begin{bmatrix} A_{cl} P + P A_{cl}^T + B_{cl} B_{cl}^T & P C_{cl}^T_c \\ C_{cl} P & -\gamma^2 I \end{bmatrix} < 0$$

$$P = P^T > 0$$

The optimization problem above is not yet convex because of the products $KP$ arising in terms like $A_{cl} P$. So, defining the variables $Y = Y^T = P$, $L = K Y$ and $W = W^T$ and using Schur's complement it is possible to rewrite the problem above as the LMI problem

$$\min \ trace (W)$$

$$\text{s.t. } \begin{bmatrix} H & Y C_{cl}^T + L^T D_{cl2}^T \\ C_{1y} + D_{12} L & -\gamma^2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} Y & Y C_{cl}^T + L^T D_{cl2}^T \\ C_{2y} + D_{22} L & W \end{bmatrix} > 0$$

where $H = AY + YA^T + B_2 L + L^T B_2^T + B_1 B_1^T$.

### 2.3. LMI formulation for regional pole constraints

In the synthesis of control systems, meeting some desired transient performance objectives (to ensure fast and well-damped transient response, reasonable feedback gain, etc.) should be considered. Generally, $H_2$-norm and $H_\infty$ synthesis design do not directly deal with the transient response of the closed-loop system. In contrast, a satisfactory transient response can be guaranteed by confining its poles in a prescribe region. For many practical problems, exact pole assignment may not be necessary; it suffices to locate the closed-loop poles in a prescribe subregion in the complex left half plane.
**Definition 1.** LMI stability region [19]. A subset $D$ of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{m \times m}$ such that

$$D = \{z \in C : f_D(z) < 0\}$$

where the characteristic function $f_D(z)$ is given by $f_D(z) = [\alpha_{kl} + \beta_{kl}z + \beta_{kl}z^\dagger]_{1 \leq k, l \leq m}$ ( $f_D$ is valued in the space of $m \times m$ Hermitian matrices).

The location of the closed-loop poles of $A + B_2K$ in Eq. (6) concern with the performance of the closed-loop system, i.e., the stability, the decay rate, the maximum overshoot, the rise time and settling time. Therefore, it is interesting work for control engineers to design the control gain $K$ such that the closed-loop poles of $(A + B_2K)$ lie in a suitable subregion of the left half plane. The interesting region for control purposes is the set $S(\alpha, r, \theta)$ of complex number $x + jy$ such that

$$x < -\alpha < 0, \; |x + jy| < r, \; \text{and} \; \tan(\theta)x < -|y|$$

as shown in Fig. 2. Confining the closed-loop poles to this region ensures a minimum decay rates $\alpha$, a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_d = r \sin \theta$ ($\theta$ in radian).
Fig. 2. Region $S(\alpha, r, \theta)$

The LMI formulations for the poles of $(A + B_2 K)$ lie in the region $S(\alpha, r, \theta)$ are characterized as the following LMIs [19,20]: if there exists symmetric $P > 0$ such that

\[
(A + B_2 K) P + P(A + B_2 K)^T + 2 \alpha P < 0
\]  
(16)

and

\[
\begin{bmatrix}
-rP & (A + B_2 K)^T P \\
P(A + B_2 K)^T & -rP
\end{bmatrix} < 0
\]  
(17)

with $L = KP$, $Y = P$, the above LMIs are equivalent to

\[
AY + YA^T + B_2 L + (B_2 L)^T + 2\alpha Y < 0
\]  
(19)

\[
\begin{bmatrix}
-rY & AY + B_2 L \\
YA^T + (B_2 L)^T & -rY
\end{bmatrix} < 0
\]  
(20)

\[
\begin{bmatrix}
\sin(\theta) (AY + B_2 L + YA^T + (B_2 L)^T) \\
\cos(\theta) (YA^T + (B_2 L)^T - AY - B_2 L) \\
\cos(\theta) (AY + B_2 L - YA^T - (B_2 L)^T) \\
\sin(\theta) (AY + B_2 L + YA^T + (B_2 L)^T)
\end{bmatrix} < 0
\]  
(21)
From the analysis above, if there exists $Y$ and $L$ for Eqs. (19)-(21), then the poles of $(A + B_2K)$ lie in the region $S(\alpha, r, \theta)$.

2.4. Multiobjective $H_2/H_\infty$ control with regional pole constraints

The combination objectives of robust multiobjective $H_2/H_\infty$ control with regional pole constraints can be characterized as follows:

$$\min_{[Y, L]} \text{trace}(W)$$

s.t. Eq. (12), Eq. (13) and Eqs. (19)-(21)

From analysis above, the most important task in this paper is to find the variable $Y$, $L$, $\gamma$ and $W$ can be solved using standard optimization techniques. Once a feasible solution $(Y, L)$ satisfying Eq. (22) is found, the required state feedback gain matrix can be computed as

$$K = LY^{-1}$$

which leads to

$$\left\| T_{czx} \right\|_\infty \leq \gamma, \left\| T_{cz} \right\|_2 \leq \sqrt{\text{trace}(W)}$$

The Lyapunov shaping paradigm for multi-objective design provides a greater flexibility than single-objective optimal design techniques such as $H_\infty$ synthesis or $H_2$-norm technique.

3. Dynamic model of the power system

In this study, a single-machine infinite-bus (SMIB) power system [21] as shown in Fig. 3 is considered. The static fast exciter is shown in Fig. 4. Block diagram of conventional power system stabilizer (CPSS) used for comparison is shown in Fig. 5. Neglecting the effect of damper winding, stator transient and resistance, the synchronous machine together with its excitation system is modeled using the following 4th order non-linear dynamic equations:

$$\omega = \frac{1}{M} (T_m - T_e + D(\omega - 1))$$

$$\dot{\delta} = \omega_p (\omega - 1)$$

$$\dot{E}_q = \frac{1}{T_d \omega_p} \left( E_{f_d} - (x_d - x_d')i_d - E_q' \right)$$
\[ \dot{E}_{fd} = \frac{1}{T_A} \left( K_A (V_{ref} - v_t + u) - E_{fd} \right) \]  

(28)

It can be seen that this model is non-linear. To permit analysis and control of the power system, the model is linearised around the operating point. The state variables of this model are $\Delta\omega, \Delta\delta, \Delta E'_q, \Delta E_{fd}$, respectively, angular speed, rotor angle, voltage behind transient, and excitation voltage. In this study, we assumed that all state variables are available for feedback. The power input to the generator shaft is assumed constant, the network is represented by a set of algebraic equations and the loads are modeled by constant impedance.

Fig. 3. A SMIB power system

Fig. 4. Static fast exciter model

Fig. 5. Block diagram of conventional PSS
4. Simulation results

A typical single-machine infinite-bus (SMIB) power system [21] is chosen for analysis of the proposed controller. The machine data and the exciter data are shown in Tables 1 and 2, respectively. The data for CPSS constants is given in Table 3.

Table 1
Machine data

<table>
<thead>
<tr>
<th>M</th>
<th>T_{d0}</th>
<th>D</th>
<th>x_d</th>
<th>x_d'</th>
<th>x_q</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.26</td>
<td>7.76</td>
<td>0</td>
<td>0.973</td>
<td>0.19</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 2
Exciter data

<table>
<thead>
<tr>
<th>K_A</th>
<th>T_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3
CPSS constants

<table>
<thead>
<tr>
<th>T_e</th>
<th>T_1</th>
<th>T_2</th>
<th>K_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.685</td>
<td>0.1</td>
<td>7.09</td>
</tr>
</tbody>
</table>

The operating condition: \( P_e = 1.0 \) pu, \( Q_e = 0.015 \) pu and \( v_t = 1.05 \) pu are chosen as the nominal operating condition and other operating points are regarded as perturbations of the nominal system. Four different loading conditions representing nominal, heavy, light, and leading power factor (PF) are considered as given in Table 4. The eigenvalues of the nominal system are \( 0.295 \pm 4.96 \) and \(-10.4 \pm 3.28 \). It is observed that the electromechanical mode (characterized by the pair of eigenvalues \( 0.295 \pm 4.96 \)) is negatively damped and the eigenvalues for this mode should be shifted leftward to more desirable locations into the left half s-plane.

The technique described in Section 2 was applied to the damping controller design for the study system. The feasibility problem was solved for \((Y,L)\) and the required state feedback matrix was obtained as \( K = LY^{-1} \), where \( Y \) is a symmetric, positive definite matrix and \( L \) is the matrix introduced to obtain linearity.

To determine the gain of the state feedback controller, the minimization
problem Eq. (22) is solved using LMI-control toolbox. We can solve the eigenvalue problem in Eq. (22) with pole constraints in the region of \( S(2,16,1.1593) \). In this case, the associated matrices and scalars are

\[
Y = \begin{bmatrix}
1.5101e-2 & -2.7299e-1 & 1.4668 & -1.0090e2 \\
-2.7299e-1 & 1.0888e1 & -1.6902e1 & 6.1811e2 \\
1.4668 & -1.6902e1 & 1.9726e2 & -1.9365e4 \\
-1.0090e2 & 6.1811e2 & -1.9365e4 & 2.5241e6
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
1.5101e-2 & -4.7418 & -2.5484e1 & 2.4869e3
\end{bmatrix}
\]

\[
\gamma = 0.8864, \quad \sqrt{\text{trace}(W)} = 1.1388
\]

and thus the feedback gain matrix is

\[
K = [288.65 \quad 1.9197 \quad -3.7586 \quad -0.01678]
\]

which makes the poles of \((A + B_2K)\) locate at \(-5.77 \pm j10.2\) and \(-12.70 \pm j5.12\). As a final check, we can calculate

\[
\left\| F_{c,w} \right\|_\infty = 0.8748, \quad \left\| F_{c,w} \right\|_2 = 0.5616
\]

that satisfies the inequality Eq. (24).

For evaluation purposes, the performance of the system with the proposed controller was compared to the CPSS and \(H_2\) control (without LMI stability region). A small disturbance of 10% step increase in the reference voltage \((V_{ref})\) was applied to the SMIB power system at four different operating conditions. The system eigenvalues and damping ratios of electromechanical modes at various operating conditions are given in Table 5. Note that the damping ratio as shown in Table 5 is written in the brackets. It is clear that the system stability is greatly enhanced with the proposed stabilizer. It can also be seen that all eigenvalues and damping ratios with the proposed stabilizer lie in an LMI region of \(S\). Simulation results shown in Figs 6-9 illustrate the performance and robustness of the proposed PSS under different operating conditions. It can be seen that the proposed PSS yields the better dynamic performance, it is less sensitive to changes in the system parameters and more robust against model uncertainties.
Table 4  
Operating conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>P (pu)</th>
<th>Q (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nominal</td>
<td>1.0</td>
<td>0.015</td>
</tr>
<tr>
<td>2. Heavy</td>
<td>1.2</td>
<td>0.20</td>
</tr>
<tr>
<td>3. Light</td>
<td>0.7</td>
<td>0.10</td>
</tr>
<tr>
<td>4. Leading PF</td>
<td>0.7</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

For completeness and verifications, all controllers were tasted at the following disturbances and loading conditions.
(a) Nominal loading \( (P,Q) = (1.0,0.015) \) pu with one-line fault.
(b) Heavy loading \( (P,Q) = (1.2,0.3) \) pu with one-line fault.

A line fault is assumed; one of the transmission lines (as shown in Fig. 3) met a line-fault and the circuit breaker operated. The simulation results for cases (a) and (b) as shown in Fig. 10 and Fig. 11, respectively. Fig. 11 shows both CPSS and \( H_2 \) control fail to stabilize the system with disturbance (b), the proposed PSS provide good damping characteristics and system is stable under this disturbance. It is clear that the proposed PSS exhibits better damping properties and guarantees robust stability of the power systems.

Table 5  
System eigenvalues and damping ratios at various operating conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>CPSS</th>
<th>( H_2 ) Control</th>
<th>Proposed PSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nominal</td>
<td>-1.52 ± j 3.17; (0.43)</td>
<td>-3.67 ± j 4.52; (0.63)</td>
<td>-5.77 ± j 10.2; (0.49)</td>
</tr>
<tr>
<td>2. Heavy</td>
<td>-1.07 ± j 3.02; (0.33)</td>
<td>-4.35 ± j 5.22; (0.64)</td>
<td>-6.32 ± j 11.3; (0.49)</td>
</tr>
<tr>
<td>3. Light</td>
<td>-1.53 ± j 3.40; (0.41)</td>
<td>-2.07 ± j 4.40; (0.43)</td>
<td>-3.64 ± j 7.17; (0.45)</td>
</tr>
<tr>
<td>4. Leading PF</td>
<td>-1.98 ± j 3.39; (0.51)</td>
<td>-2.28 ± j 5.41; (0.39)</td>
<td>-4.05 ± j 8.45; (0.43)</td>
</tr>
</tbody>
</table>
Fig. 6. Response with 10% step in $V_{ref}$ for case 1

Fig. 7. Response with 10% step in $V_{ref}$ for case 2

Fig. 8. Response with 10% step in $V_{ref}$ for case 3
Fig. 9. Response with 10% step in $V_{\text{ref}}$ for case 4

Fig. 10. Response with fault disturbance for case (a)

Fig. 11. Response with fault disturbance for case (b)
5. Conclusions

This paper has presented the design of multiobjective $H_2/H_\infty$ control with regional pole constraints for damping power system oscillations. The required state feedback gain has been obtained by solving a linear matrix inequality (LMI) feasibility problem that robustly assigns the closed-loop poles in a prescribed LMI region. The performance of the proposed stabilizer on a SMIB power system is seen to be robust over a wide range of operating conditions. Finally, simulation results show the effectiveness and robustness of the proposed stabilizer to enhance the damping of low frequency oscillations.

References


